Minutes of the Meeting on Multiscale Modelling at EPFL, Lausanne, July 14-15 2004

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Introduction

This meeting was held at Ecole Polytechnique Fédérale de Lausanne on July 14th-15th, 2004. It concerned mainly problems of multiscale modelling and their implementation in LifeV.

1 People

The following people were involved in the meeting:

- Luca Formaggia, MOX, Italy.
- Vincent Martin, MOX, Italy.
- Vuk Milisic, EPFL, Switzerland.
- Martin Prosi, MOX, Italy.
- Christophe Prud'homme, LifeV Coordinator, EPFL, Switzerland.

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New people

Two students will soon start developping in LifeV at MOX:

- Christian Vergara (Navier Stokes 3D: taking into account boundary condition prescribed as averaged pressure or velocities).
- Alexandra Moura (3D compliant coupled to 0D).

2 Multiscale blood flow models

2.1 Modelling the graph of the multiscale blood flow

Input files: prescribing the multiscale problem

It was decided to use a `XML` typology to describe entirely the multiscale problem. This should be an easy way of setting the problem to be solved (with, hopefully, a Graphic User Interface).

Each simulation will be completely described (*data*: physical, numerical, geometrical, ... parameters), and the *graph* of the simulations, i.e. which vessel communicates with which vessels, will also be fully described.

Graph

The type of implementation of the arterial tree was discussed. A deeper look at the C++ `boost` graph library is necessary to know if we should use or not this library as a basis for our implementation.

It seems that the representation of an arterial tree should be as follow (see Figure 1):

- Edge of the graph: a numerical simulation. For instance: 0D, 1D linear, 1D non-linear, 3D...
- Node of the graph: an interface that treats the coupling between two, or more simulations.

On Figure 1, a simple example is shown. The heart model is denoted by H, and the final termination by T. They are a simple Interface/Node, communicating with only one simulation/Edge. The Interfaces/Nodes $Ij, j = 1, \ldots, 5$ are interfaces between simulations/Edges. For instance $I4$ allows to couple one 1D simulation (inlet) to two 1D simulations outlet; consequently it represents physically a bifurcation. As $I2$ couples a 3D simulation to two 1D simulation, this means that the 3D simulation has two outlets, and therefore involves a bifurcation.

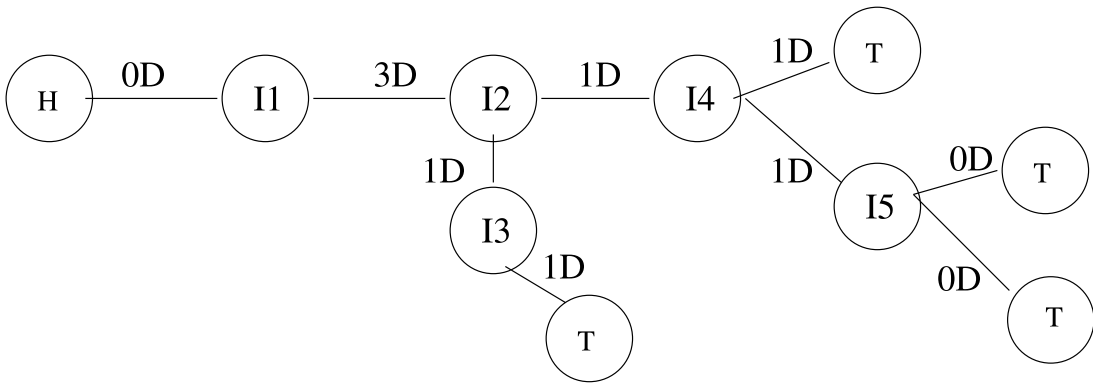

Figure 1: Example of a graph representing a multiscale blood flow model.

2.2 Implementing the Simulations/Edges

A recent version of the 1D non-linear model with a second order Taylor-Galerkin scheme was programmed by Daniele Lamponi in Fortran90. It will be reprogrammed in C++ in LifeV by Vincent Martin, in order to be consistent with the informatical implementation of the multiscale that is to be designed.

The 1D linear model was implemented by Vuk Milisic in Fortran90 with a MUSCL scheme.

A version of a 0D model was also implemented by Vuk Milisic. The implementation of the 0D simulations in the graph was discussed: should each windkessel element be one edge of the graph? Or should a 0D-edge be a black box representing an entire block of 0D elements? This question is not yet settled.

2.3 Modelling the Interfaces/Nodes

The implementation of the Interfaces/Nodes were discussed. The particular example of nodes coupling two 1D vessels, or one 1D and one 0D simulations, were taken. Each Interface/Node should perform the computations necessary for each Simulations/Edges with which it communicates.

For instance, in the case of two 1D models, at each time step n, the Interface/Node should receive some functions, and some values from the two 1D models and return the boundary values at time $n+1$. More precisely, the following non-linear system for the interface variables A^+, A^-, Q^+ and

Q^- at time step $n+1$ must be solved:

$$\begin{cases} Q^- - Q^+ = 0 \\ \psi(A^-; A_0^-, \beta_0^-) + \frac{\rho}{2}\left(\frac{Q^-}{A^-}\right)^2 - \psi(A^+; A_0^+, \beta_0^+) + \frac{\rho}{2}\left(\frac{Q^+}{A^+}\right)^2 = 0 \\ W_1^-(A^-, Q^-) - W_1^- = 0 \\ W_2^+(A^+, Q^+) - W_2^+ = 0 \ , \end{cases} \tag{1}$$

where $\psi(\cdot), W_j^i(\cdot),\ i = +,-,\ j = 1,2$ are given functions, and $\beta, \rho, W_j^i,\ i = +,-,\ j = 1,2$ are given values. These functions and their gradients, and these values must be transmitted to the Interface/Node, that performs a Newton solve to compute A^+, A^-, Q^+ and Q^-, and then returns A^+, Q^+ to one vessel and A^-, Q^- to the other.